Hypothesis

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Definition 1. `Set`

A set is a collection of elements of some type . In probability theory, a set is an *event*.

Definition 2. `Subtype`

All the elements of a type that satisfy a predicate.

Definition 3. `Set.univ`

The universal set on a type   is the set containing all elements of . It plays the role of *state space* or *sample space* in probability theory.

Definition 4. `Bool`

The Boolean values, true and false. Can also be used to represent "heads and tails" in probability theory.

Definition 5. `Prod`

The product type, usually written   × . Product types are also called pair or tuple types. Elements of this type are pairs in which the first element is an   and the second element is a .

For example, Bool × Bool can be used to represent two successive tosses of a coin.

Lemma 6. *`Prod.ext`*

The condition for two pairs to be equal.

Lemma 7. *`Set.ext`*

The condition for two sets to be equal.

Definition 8. `Set.Icc`

Icc a b is the left-closed right-closed interval $[a, b]$. For example, taking $a = 1 \in \mathbb{N}$ and $b = 6$ we get the possible outcomes from tossing a die.

Definition 9. `Nat`

The natural numbers, starting at zero.

Definition 10. `Real`

The type $\mathbb{R}$ of real numbers constructed as equivalence classes of Cauchy sequences of rational numbers.

Definition 11. `NNReal`

Nonnegative real numbers, denoted as 0 within the NNReal namespace.

Definition 12. `WithTop`

Attach $\top$ to a type. For example $\top$ could be $+\infty$, a possible value of the measure of a set. `WithTop` is superficially the same as `Option`.

Definition 13. `ENNReal`

The extended nonnegative real numbers. This is usually denoted $[0, \infty]$, and is relevant as the codomain of a measure.

Definition 14. `Set.compl`

The complement of a set s is the set of elements not contained in s.

Note that you should not use this definition directly, but instead write s^c.

Definition 15. `Set.inter`

The intersection of two sets s and t is the set of elements contained in both s and t.

Note that you should not use this definition directly, but instead write s t.

Definition 16. `Set.union`

The union of two sets s and t is the set of elements contained in either s or t.

Note that you should not use this definition directly, but instead write s t.

Definition 17. `Set.singleton`

The singleton of an element a is the set with a as a single element.

Note that you should not use this definition directly, but instead write $\{a\}$.

In probability theory, a is an outcome and $\{a\}$ is an event.

Definition 18. `Set.powerset`

 s is the set of all subsets of s.

In probability theory, this may be the family of all events; or some subsets may fail to be events.

Definition 19. `Set.iUnion` Indexed union of a family of sets.

Definition 20. `MeasurableSpace`

A measurable space is a space equipped with a -algebra:

MeasurableSet (the sets in the σ-algebra $\mathcal{A}$)

MeasurableSet.empty ($\emptyset \in \mathcal{A}$)

MeasurableSet.compl ($\mathcal{A}$ is closed under complement)

MeasurableSet.iUnion ($\mathcal{A}$ is closed under countable union, indexed by $\mathbb{N}$)

Definition 21. `Measure`

A measure is defined to be an outer measure that is countably additive on measurable sets, with the additional assumption that the outer measure is the canonical extension of the restricted measure.

The measure of a set s, denoted s, is an extended nonnegative real. The real-valued version is written .real s.

Definition 22. `ProbabilityMeasure`

Probability measures are defined as the subtype of measures that have the property of being probability measures (i.e., their total mass is one).

Definition 23. `Measure.map`

The pushforward of a measure. It is defined to be 0 if f is not an almost everywhere measurable function.

For the pushforward of m along f, the outer measure on s is defined to be m (f ¹' s).

Definition 24. `ProbabilityMeasure.map`

The pushforward of a probability measure.

It is suggested on page 4 of Jacod and Protter.

Definition 25. `PMF` Probability mass function.

Definition 26. `PMF.uniformOfFintype` Uniform distribution, as in the Example on page 5 of Jacod and Protter.

0.1 Chapter 2

Definition 27. `MeasurableSpace.generateFrom` The σ-algebra generated by a family of sets.

Definition 28. `MeasurableSpace.generateFrom` The σ-algebra generated by a singleton. Not sure why it requires special mention.

The following results say that the trivial σ-algebra $\bot$ can be generated in three ways: `MeasurableSpace.generateFrom_empty`, `MeasurableSpace.generateFrom_singleton_empty`, `MeasurableSpace.generateFrom_singleton_univ`.

The powerset as a σ-algebra is $\top$, for example ($\top$: MeasurableSpace $\mathbb{R}$).

Definition 29. `TopologicalSpace`
A topology on a given type.

Definition 30. `borel`
MeasurableSpace structure generated by TopologicalSpace.

Theorem 31. *`Real.borel_eq_generateFrom_Iic_rat`*
Theorem 2.1 (page 8) in Jacod and Protter.

The fact that a measure is finitely additive is `measure_union`. The next property mentioned in JP is `MeasureTheory.Measure.mono`.

Theorem 2.3 is `MeasureTheory.measure_biUnion_eq_iSup`

Set indicator: `Set.indicator`