Dyadic Deontic Logic

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Definition 1. Axiom 5(a) for a function ob : $\mathcal{P}(U) \to \mathcal{P}(\mathcal{P}(U))$ says that $\emptyset \notin \mathrm{ob}(X)$ for all $X \in \mathcal{P}(U)$.

Definition 2. Axiom 5(b) for a function ob : $\mathcal{P}(U) \to \mathcal{P}(\mathcal{P}(U))$ says that

$$\forall (XYZ : SetU), Z \cap X = Y \cap X \to (Z \in obX \leftrightarrow Y \in obX)$$

Definition 3. Axiom 5(d) for a function ob : $\mathcal{P}(U) \to \mathcal{P}(\mathcal{P}(U))$ says that

$$\forall (XYZ : SetU), Y \subseteq X \to Y \in obX \to X \subseteq Z \to Z\ X \cup Y \in obZ$$

Definition 4. Axiom 5(bd) for a function ob : $\mathcal{P}(U) \to \mathcal{P}(\mathcal{P}(U))$ says that $\forall (XYZ \in \mathcal{P}U), Y \in \mathrm{ob}\, X \wedge X \subseteq Z \to Z \setminus X \cup Y \in \mathrm{ob}\, Z$.

Theorem 5. *If* ob *satisfies 5(b) and 5(d) then it satisfies 5(bd).*